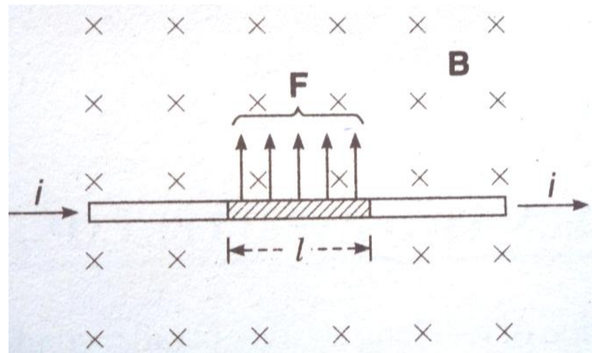
Force on a current carrying conductor

Consider a conductor of length l is placed perpendicular to the magnetic field of induction B. Magnetic field is perpendicular to the plane of the paper. Let i be the current flowing through the conductor in time t (as shown in the figure).

We know that the force F acting on a charge q in motion with velocity V is given by



$$\boldsymbol{F} = q(\boldsymbol{V} \, x \, \boldsymbol{B})$$

$$\boldsymbol{F} = q(\frac{l}{t} \, x \, \boldsymbol{B})$$

$$\boldsymbol{F} = \frac{q}{t}(\boldsymbol{l} \, x \, \boldsymbol{B})$$

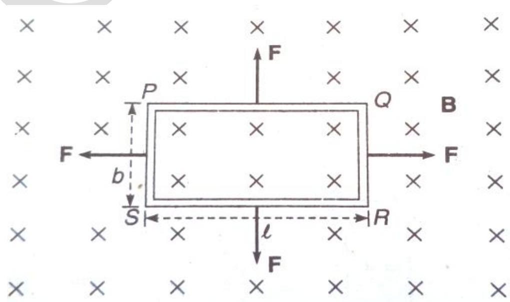
But $\frac{q}{t} = i \;\; current$

So $\boxed{\boldsymbol{F} = i(\boldsymbol{l} \, x \, \boldsymbol{B})}$

or $\boxed{\boldsymbol{F} = Bil \, Sin\theta}$

Torque on a current carrying loop

Consider a loop P Q R S of length l and breadth b carrying current i and it is placed in a uniform magnetic field of induction B.

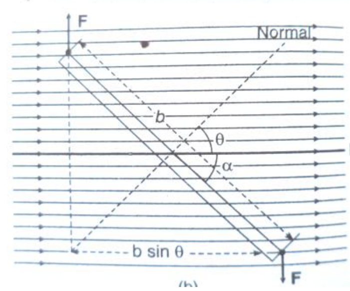


PQ = RS = l and PS = QR = b

As the sides PQ and RS are perpendicular to the field B.

The forces acting on each side PQ & RS is $\boldsymbol{F} = Bil$

The current in these two edges are in opposite direction. So these two forces are in opposite direction and constitute a couple or torque.



Let θ be the angle between the magnetic field B and the normal drawn to the plane of the loop.

Then torque is given by τ = Force X Perpendicular distance

$$\tau = Bil \,.\, b \; Sin\,\theta \quad \text{or} \quad \tau = Bi(l \,.\, b) \; Sin\,\theta$$

$$\tau = Bi \; A \; Sin\,\theta$$

∵ l.b = A = Area of the loop.

If the loop or coil has N no. of turns, then the torque is given by

$$\boxed{\tau = NBi \; A \; Sin\,\theta}$$

This principle is used in moving coil galvanometer or ballistic galvanometer.

Biot-savart's law

This law gives the magnetic induction B around the current carrying conductor.

Consider an irregular shaped conductor AB carrying current i. Let dB be the magnetic induction at P due to the small element of length dl. The point P is at a distance 'r' from the small element. Let θ be the angle between **r** and **dl.**

Magnetic induction dB is directly proportional to the current i flowing through the conductor. $dB \propto i$

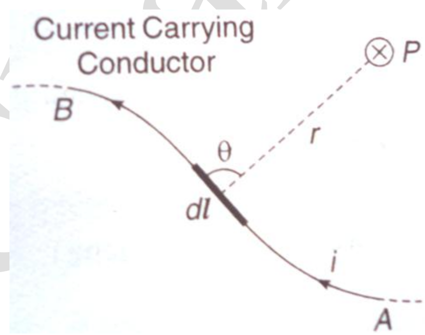
dB is directly proportional to the length of the element dl.

$$dB \propto dl$$

dB is also directly proportional to the sine of the angle θ between dl and r.

$$dB \propto Sin\,\theta$$

dB is inversely proportional to the square of the distance from the point P to the element dl.



$$dB \propto \frac{1}{r^2}$$

Or $$dB \propto \frac{i\,dl\,Sin\theta}{r^2}$$

Or $$dB = \frac{\mu_o}{4\pi}\frac{i\,dl\,Sin\theta}{r^2}$$

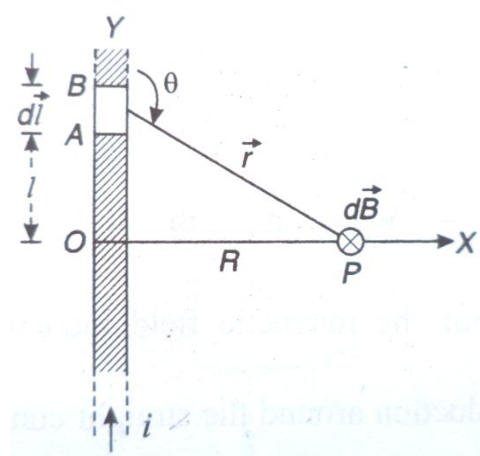
Here $\frac{\mu_o}{4\pi} = Proportionality\;constant$

μ_o = Permeability of the free space.

If $\hat{r}$ = Vector along r

Then, in vector form $$\overline{dB} = \frac{\mu_o}{4\pi}\frac{i\,\overline{\mathbf{dl}}\,x\,\hat{r}\,Sin\theta}{r^3}$$

Magnetic induction B due to long straight conductor carrying current



Consider a long straight conductor of infinite length carrying current i. Let P be the point where the magnetic induction is to be measured. This point is at a perpendicular distance R from the conductor. Take a small element AB of length dl at a distance r from the point P. The angle between dl and r be θ in the clockwise direction.

As per Biot-Savart's law the magnetic induction at P due to the small element dl is given by

$$dB = \frac{\mu_o}{4\pi}\frac{i\,dl\,Sin\theta}{r^2}$$

Induction due to the whole conductor $B = \int dB = \int_{-\infty}^{\infty}\frac{\mu_o}{4\pi}\frac{i\,dl\,Sin\theta}{r^2} \longrightarrow (1)$

From figure $r = (l^2 + R^2)^{1/2}$ & $Sin\,\theta = Sin\,(\pi - \theta) = \frac{R}{r} = \frac{R}{(l^2+R^2)^{1/2}}$

Substituting the values of r & Sin θ in eqn. (1)

Then $B = \frac{\mu_o i}{4\pi} \int_{-\infty}^{\infty} \frac{R\, dl}{(l^2 + R^2)^{3/2}}$

If α is the angle between r and R

Then $\tan\alpha = \frac{l}{R}$ or $l = R\tan\alpha$

On differentiation $dl = R\, sec^2\alpha\, d\alpha$

Converting the integration limits in terms of angle

$$B = \frac{\mu_o i}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{R. R\, sec^2\alpha\, d\alpha}{R^3(1 + tan^2\alpha)^{3/2}}$$

$$B = \frac{\mu_o i}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{d\alpha}{R .\sec\alpha}$$

$\because (1 + tan^2\alpha) = sec^2\alpha$

$$B = \frac{\mu_o i}{4\pi R} \int_{-\pi/2}^{\pi/2} \cos\alpha\, d\alpha$$

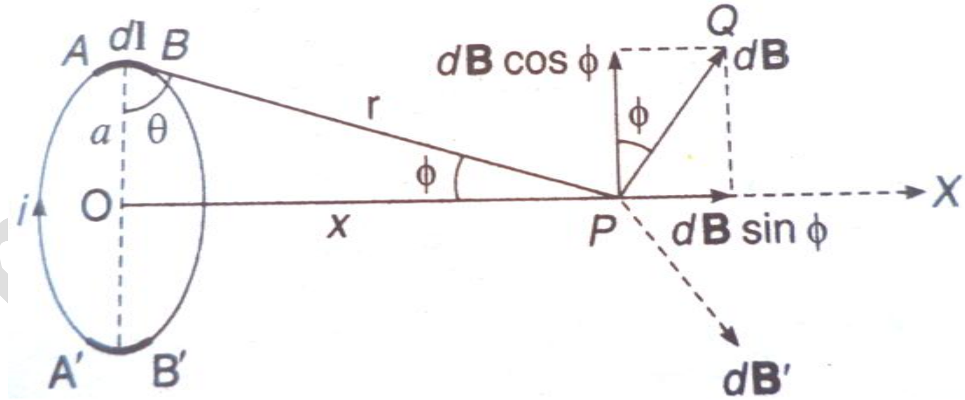
$$B = \frac{\mu_o i}{4\pi R} [\sin\alpha]_{-\pi/2}^{\pi/2}$$

$$B = \frac{\mu_o i}{4\pi R} (1 + 1)$$

So the equation for magnetic induction is $B = \frac{\mu_o i}{2\pi R}$

Magnetic induction on the axis of a circular loop carrying current

Consider a circular coil of radius 'a' carrying current 'I'. Let P be a point where the magnetic induction is to be measured. It is on the axis of the coil at distance 'x' from the centre of the coil.



Consider a small element AB of length dl in the coil and it is at a distance r from the point P. The angle between dl and r is θ = 90^0.

Then the magnetic induction dB due to the element dl at p is given by Biot- Savart's law.

$$dB = \frac{\mu_o}{4\pi} \frac{i\, dl\, Sin90^0}{r^2} = \frac{\mu_o}{4\pi} . \frac{i\, dl}{r^2}$$

The angle between r and axis of the coil is ϕ.

$\overline{dB}$ is in perpendicular direction to the plane containing $\bar{r}$ and $\overline{dl}$. This vector $\overline{dB}$ can be resolved into two perpendicular components, one (dB sinϕ) along axis and the other (dB cosϕ) perpendicular to the axis.

- Take another element A^1B^1 in the coil diametrical opposite to AB. The magnetic induction dB at P due to this element A^1B^1 can also be resolved into two perpendicular components.
- The component along the axis (dB sinϕ) can be added to the previous one. But the component perpendicular to the axis (dB cosϕ) is quite opposite to the previous perpendicular component & they cancel each other.
- Like this, the resultant perpendicular component comes out as zero and only the component along the axis exists.

Magnetic induction along the axis $B = \int dB \sin \emptyset$

$$B = \frac{\mu_o i}{4\pi r^2}\int dl \sin \emptyset$$

$$B = \frac{\mu_o i}{4\pi r^2}\int dl \,. \left(\frac{a}{r}\right) \qquad \because \sin \emptyset = \left(\frac{a}{r}\right)$$

$$B = \frac{\mu_o ia}{4\pi r^3}\int dl \,. \longrightarrow (1)$$

$\int dl = 2\pi a =$ Circumference of the coil. $\longrightarrow$ (2)

From figure $r = (a^2 + x^2)^{1/2}$

Substituting eqn. (2) in eqn. (1)

$$B = \frac{\mu_o \, i \, a}{4\pi(a^2+x^2)^{3/2}} \, x \, 2\pi a = \frac{\mu_o \, i \, a^2}{2(a^2+x^2)^{3/2}}$$

If the coil has N no. of turns. Then $\boxed{B = \frac{\mu_o N \, i \, a^2}{2(a^2+x^2)^{3/2}}}$ This is along the axis of the coil.

Case 1:- At the centre of the coil x = 0

$$\boxed{B = \frac{\mu_o N \, i \, a^2}{2a^3} = \frac{\mu_o N \, i}{2a}}$$

Case 2:- At far away from the centre of the coil $x >> a$ & $(a^2 + x^2)^{3/2} \approx x^3$

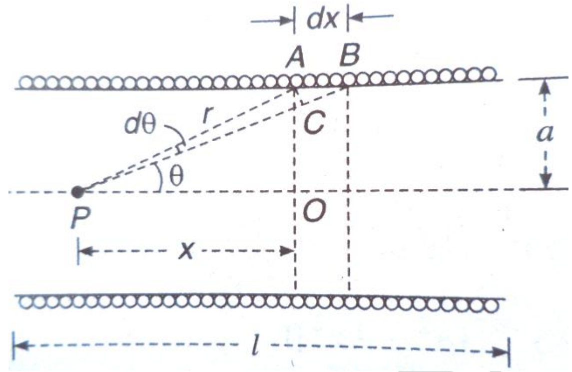
$$\boxed{B = \frac{\mu_o N \, i \, a^2}{2x^3}}$$

Magnetic induction due to solenoid

Consider a solenoid of length 'l' carrying current 'i' and its radius is 'a'. The total no. of turns is 'N' and number of terms per metre is 'n'. So n = N/l

Here three different cases arise 1) Field at inside point. 2) Field at an axial endpoint. 3) Field at the centre of the solenoid of finite length

1) Field at inside point.



- ✓ Let P be the point on the axis where the field is to be measured. Consider a small elementary part of the solenoid AB of width 'dx'.
- ✓ The point P is at a distance 'r' from the elementary part and at a distance 'x' from the centre of the elementary part.
- ✓ The angle between r and axis of the coil is θ and the angle made by the width of the elementary part at P is dθ.

The magnetic induction dB due to the elementary part is

$$dB = \frac{\mu_o (n\,dx)\, i\, a^2}{2(a^2 + x^2)^{3/2}} \longrightarrow (1)$$ as per the mag. induction of the circular coil.

From the figure $\quad AC = r\, d\theta$

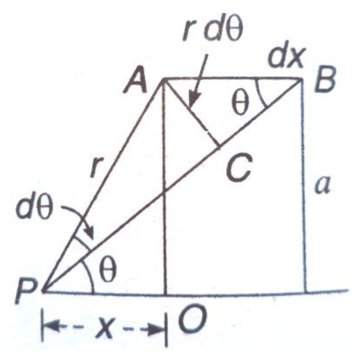
From the $\Delta ABC \quad \sin\theta = \frac{r\, d\theta}{dx} \quad (or) \quad dx = \frac{r\, d\theta}{\sin\theta}$

From the $\Delta APO \quad a^2 + x^2 = r^2 \quad \therefore (a^2 + x^2)^{3/2} = r^3 \longrightarrow$ (2)

Substituting eqn. (2) in (1)

$$dB = \frac{\mu_o (n \frac{r\, d\theta}{\sin\theta})\, i\, a^2}{2r^3}$$

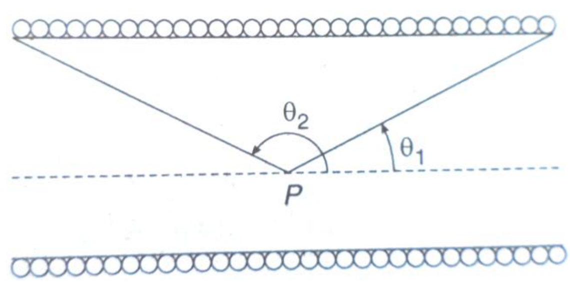
$$dB = \frac{\mu_o\, n\, d\theta\;\, i}{2\sin\theta}\left(\frac{a}{r}\right)^2$$



But $\quad \sin\theta = \frac{a}{r} \quad$ or $\quad \left(\frac{a}{r}\right)^2 = sin^2\theta$

$$dB = \frac{\mu_o\, n\, d\theta\;\, i}{2\sin\theta}(\sin\theta)^2$$

$$\boxed{dB = \frac{\mu_o\, n\, d\theta\;\, i\, \sin\theta}{2}}$$



$$B = \int_{\theta_1}^{\theta_2} dB = \int_{\theta_1}^{\theta_2} \frac{\mu_o\, n\;\, i\;\, \sin\theta\, d\theta}{2}$$

$$B = \frac{\mu_o\, n\, i}{2}\,[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$\boxed{B = \frac{\mu_o\, n\, i}{2}\,[\cos\theta_1 - \cos\theta_2]} \longrightarrow (3)$$ This is general equation for B

1) <u>For the field at the centre of solenoid of infinite length</u>, the limits are $\theta_1 = 0$ and $\theta_2 = \pi$

$$B = \frac{\mu_o\, n\, i}{2}\,[\cos 0 - \cos\pi] \quad \text{(or)} \quad B = \frac{\mu_o\, n\, i}{2}\,[1 - (-1)]$$

$$\boxed{B = \mu_o\, n\, i}$$

2) For the field at one axial end of solenoid of infinite length, the limits are $\theta_1 = 0$ and $\theta_2 = \pi/2$

$$B = \frac{\mu_o\, n\, i}{2}\,[\cos 0 - \cos \pi/2] \qquad \text{(or)} \qquad B = \frac{\mu_o\, n\, i}{2}\,[1 - (0)]$$

$$\boxed{B = \frac{\mu_o\, n\, i}{2}}$$

3) For the field at the centre of solenoid of finite length,

Let 'l' be the length of the solenoid and the distance from the centre on either side is l/2.

$$\therefore \cos\theta_1 = \frac{l/2}{\left\{a^2 + \left(\frac{l}{2}\right)^2\right\}^{\frac{1}{2}}} = \frac{l}{\{4a^2 + l^2\}^{1/2}}$$

$$\& \qquad \cos(\pi - \theta_2) = \frac{-l/2}{\left\{a^2 + \left(\frac{l}{2}\right)^2\right\}^{\frac{1}{2}}} = \frac{-l}{\{4a^2 + l^2\}^{1/2}}$$

$$\therefore \cos\theta_2 = \frac{-l}{\{4a^2 + l^2\}^{1/2}}$$

Substituting these cosine values in eqn. (3)

$$B = \frac{\mu_o\, n\, i}{2}\left[\frac{l}{\{4a^2 + l^2\}^{1/2}} - \frac{-l}{\{4a^2 + l^2\}^{1/2}}\right]$$

$$B = \frac{\mu_o\, n\, i}{2}\left[\frac{l}{\{4a^2 + l^2\}^{1/2}} + \frac{l}{\{4a^2 + l^2\}^{1/2}}\right]$$

$$B = \frac{\mu_o\, n\, i}{2}\left[\frac{2l}{\{4a^2 + l^2\}^{1/2}}\right]$$

$$B = \left[\frac{\mu_o\, n\, il}{\{4a^2 + l^2\}^{1/2}}\right]$$

$$\boxed{B = \left[\frac{\mu_o\, N\, i}{\{4a^2 + l^2\}^{1/2}}\right]} \qquad \because nl = N$$

Courtesy: P.S.Brahmachary
Lecturer in Physics